2 задание 8 Вариант

$q \Rightarrow \xi$

$$\int_0^1 \frac{x^{p-1}-x^{-p}}{1-x}\,dx \overset{?}{=} \lim_{\xi\to 0}\int_0^1 \frac{x^{p-1}}{(1-x)^{1-\xi}}\,dx - \lim_{\xi\to 0}\int_0^1 \frac{x^{-p}}{(1-x)^{1-\xi}}\,dx \;=$$

$\tilde{p} = 1-p$

$$* \quad B(p,q) = \int_0^1 x^{p-1}(1-x)^{q-1}$$

$$B(\tilde{p},q) = \int_0^1 x^{\tilde{p}-1}(1-x)^{q-1}$$

$q=0$

$\xi>0$

$0<p<1$

$$= \lim_{\xi\to 0}\left(B(p,\xi) - B(1-p,\xi)\right) =$$

$$B(p,q) = \frac{\Gamma(p)\Gamma(q)}{\Gamma(p+q)}$$

$$= \lim_{\xi\to 0}\left(\frac{\Gamma(p)\Gamma(\xi)}{\Gamma(p+\xi)} - \frac{\Gamma(1-p)\Gamma(\xi)}{\Gamma(1-p+\xi)}\right) =$$

$$= \lim_{\xi\to 0}\Gamma(\xi)\left(\frac{\Gamma(p)\Gamma(1-p+\xi)-\Gamma(1-p)\Gamma(p+\xi)}{\Gamma(p+\xi)\,\Gamma(1-p+\xi)}\right) =$$

$$= \lim_{\xi\to 0}\Gamma(\xi)\Gamma(1-\xi)\left(\frac{\Gamma(p)\Gamma(1-p+\xi)-\Gamma(1-p)\Gamma(p+\xi)}{\underbrace{\Gamma(p+\xi)\,\Gamma(1-p+\xi)}_{\to 0}\;\underbrace{\Gamma(1-\xi)}_{\to 0}}\right) =$$

$$* \quad \underbrace{\Gamma(p)\,\Gamma(1-p)}\; = \frac{\pi}{\sin \pi p} \quad \text{при } 0<p<1$$

$$= \frac{\sin \pi p}{\pi}\lim_{\xi\to 0}\frac{\pi}{\sin \pi\xi}\left(\Gamma(p)\Gamma(1-p+\xi) - \Gamma(1-p)\Gamma(p+\xi)\right)$$

$$= \sin\pi p \lim_{\xi\to 0} \frac{\Gamma(p)\,\Gamma'_\xi(1-p+\xi) - \Gamma(1-p)\,\Gamma'_\xi(p+\xi)}{\pi\cos\pi\xi} \stackrel{?}{=}$$

$$= \frac{\sin\pi p}{\pi}\Big(\underbrace{\Gamma(p)\,\Gamma'_{1-p}(1-p) - \Gamma(1-p)\,\Gamma'_p(p)}_{-\frac{d}{dp}\left(\underbrace{\Gamma(p)\Gamma(1-p)}_{\frac{\pi}{\sin p\pi}}\right)}\Big) = \quad ?$$

$$= \frac{\sin\pi p}{\pi}\left(-\frac{d}{dp}\left(\frac{\pi}{\sin p\pi}\right)\right) = \frac{\sin\pi p}{\pi}\,\frac{\pi^2\cos\pi p}{\sin^2\pi p} =$$

$$= \pi\,\mathrm{ctg}(p\pi)$$

$0<p<1$

$$\lim_{\xi \to 0} \frac{\Gamma(p)\,\Gamma'_{\xi}(1-p+\xi) - \Gamma(1-p)\,\Gamma'_{p}(p+\xi)}{\pi \cos \pi \xi} = $$

$$\Gamma'_{\xi}(1-p+\xi) = -\Gamma'_{p}(1-p+\xi)$$

$$\Gamma'_{\xi}(p+\xi) = \Gamma'_{p}(p+\xi)$$

$$= \lim_{\xi \to 0} \frac{-\Gamma(p)\,\Gamma'_{p}(1-p+\xi) - \Gamma(1-p)\,\Gamma'_{p}(p+\xi)}{\pi \cos \pi \xi} =$$

$$= \cancel{\frac{d}{dp}} = -\frac{d}{dp}\left(\frac{1}{\pi}\Gamma(p)\,\Gamma(1-p)\right)$$

3 задание

16 вариант

$$\int_0^\infty \frac{x^{16}}{(x+2)^{19}}\,dx =$$

$$\int_0^\infty f(x)\,G(x) = \int_0^\infty g(y)\,F(y)\,dy$$

$$F(p) = \frac{n!}{(p-a)^{n+1}} \Rightarrow f(t) = e^{at}\cdot t^n$$ (теорема смещения)

$$f(t) = t^n \Rightarrow F(p) = \frac{n!}{p^{n+1}}$$

$$= \int_0^\infty \frac{16!}{y^{16}}\,\frac{y^{18}}{18!}\,e^{-2y}\,dy =$$

$$= \frac{1}{16\cdot 17\cdot 18}\int_0^\infty y^2 e^{-2y}\,dy = \frac{1}{16\cdot 17\cdot 18}\cdot\frac{1}{4} = \frac{1}{19584}$$

$$\int_0^\infty y^2 e^{-2y}\,dy = \left\{\begin{array}{l} y^2 = u \\ e^{-2y}dy = dv \end{array}\right\} =$$

$$= -\frac{y^2 e^{-2y}}{2}\Big|_0^\infty + \int_0^\infty e^{-2y}\,y\,dy =$$

$$= -\frac{y^2 e^{-2y}}{2}\Big|_0^\infty - \frac{y e^{-2y}}{2}\Big|_0^\infty - \frac{1}{4}e^{-2y}\Big|_0^\infty = \frac{1}{4}$$

(+)

Задание №1.

$\underline{\text{II}}$ вариант

$$\int_c^d dy \int_a^b f(x,y)\,dx = \int_a^b dx \int_c^d f(x,y)\,dy$$

$$I = \int_0^{\infty} \frac{x^2-a^2}{x^2+a^2}\,\frac{\sin x}{x}\,dx \;=$$

$$\frac{\sin x}{x} = \int_0^1 \cos yx\,dy$$

$$= \int_0^{\infty} \left(1 - \frac{2a^2}{x^2+a^2}\right) \frac{\sin x}{x}\,dx =$$

$$= \underbrace{\int_0^{\infty} \frac{\sin x}{x}\,dx}_{=\frac{\pi}{2}} - \int_0^{\infty} \frac{2a^2}{x^2+a^2}\,\frac{\sin x}{x}\,dx$$

$y \in [0, 1]$

$$\int_0^{\infty} dx \int_0^1 \frac{2a^2}{x^2+a^2} \cos yx\,dy \quad ? \quad \underline{\int_0^{\infty} \frac{2a^2}{x^2+a^2} \cos yx\,dx}$$

должен равном сходится

по признаку Вейерштрасса

1) $\left| \frac{2a^2}{x^2+a^2} \cos yx \right| \leq \frac{2a^2}{x^2+a^2}$ и $\int_0^{\infty} \frac{2a^2}{x^2+a^2}\,dx$ − сходится

и

2) $f(x,y)$ − ~~равно~~ непрерывна в Π_0 $\quad 2a^2 \frac{\pi}{2a} = a\pi$

равномерно сходится

Тогда

$$\int_0^\infty dx \int_0^1 \frac{2a^2}{x^2+a^2} \cos yx \, dy = \int_0^1 dy \int_0^\infty \frac{2a^2}{x^2+a^2} \cos yx \, dx$$

$$I_1 = \int_0^\infty \frac{\cos yx}{x^2+a^2} dx \quad \text{при } a>0$$

$$I_1 = \frac{1}{2} \operatorname{Re} \oint_C \frac{e^{iyz}}{z^2+a^2} dz$$

$z = \pm ia$ - полюса первого порядка

$$\operatorname{res} f(ia) = \lim_{z\to ia} \frac{(z-ia)\, e^{iyz}}{(z-ia)(z+ia)} =$$

? $a>0$

$$= \frac{e^{-ay}}{2ia}$$

$$I_1 = \frac{1}{2} \operatorname{Re} 2\pi i \frac{e^{-ay}}{2ia} = \frac{\pi e^{-ay}}{2a}$$

$$2a^2 \int_0^1 dy \int_0^\infty \frac{\cos yx}{x^2+a^2} dx = 2a^2 \int_0^1 \frac{\pi e^{-ay}}{2a} dy =$$

$$= -\frac{\pi}{2a}(e^{-a}-1)\, 2a^2 = -\pi(e^{-a}-1)$$

Тогда

$$I = \frac{\pi}{2} + \pi(e^{-a}-1) = \quad a>0?$$

$$= \underline{\pi\left(e^{-a}-\frac{1}{2}\right)}, \quad a>0$$

при $a < 0$

$$I_1 = \frac{1}{2} \operatorname{Re} \oint_C \frac{e^{iyz}}{z^2+a^2}\,dz$$

$z = \pm ia$

$$\operatorname{res} f(-ia) = \lim_{z \to -ia} \frac{(z+ia)\,e^{iyz}}{(z+ia)(z-ia)} =$$

$$= \frac{e^{ay}}{-2ia}$$

$$I_1 = \frac{1}{2} \operatorname{Re} 2\pi i \frac{e^{+ay}}{-2ia} = \frac{\pi e^{ay}}{-2a}$$

$$2a^2 \int_0^1 dy \int_0^\infty \frac{\cos yx}{x^2+a^2}\,dx = 2a^2 \int_0^1 \frac{\pi e^{at}}{-2a} =$$

$$= -\pi (e^{a} - 1)$$

$$\underline{I = \pi\left(e^{a} - \frac{1}{2}\right),} \quad a < 0$$

при $a = 0$

$$\underline{\int_0^\infty \frac{x^2-a^2}{x^2+a^2}\,\frac{\sin x}{x} = \frac{\pi}{2}.}$$

[redacted]

№1

④ $\int\limits_0^{\frac{\pi}{2}} \ln(a^2+b^2 tg^2 x)\,dx$

$$\int\limits_0^{\frac{\pi}{2}} \frac{tg^2 x\,dx}{\frac{a^2}{b^2}+tg^2 x} = \int\limits_0^{\frac{\pi}{2}} \left(\frac{tg^2 x+\frac{a^2}{b^2}}{tg^2 x+\frac{a^2}{b^2}} - \frac{\frac{a^2}{b^2}}{tg^2 x+\frac{a^2}{b^2}}\right)dx$$

$$= \underbrace{\int\limits_0^{\frac{\pi}{2}} 1\,dx}_{\text{сходится}} - \underbrace{\frac{a^2}{b^2}\int\limits_0^{\frac{\pi}{2}} \frac{1}{tg^2 x+\frac{a^2}{b^2}}\,dx}_{\text{непрерывная ф-ция при } b}$$

и $\Rightarrow$ значит сходится на b

РАВНОМЕРНО СХОДИТСЯ

- $\lim\limits_{x\to 0}(\ln(a^2+b^2 tg^2 x)) = \ln(\lim\limits_{x\to 0}(a^2)+\lim\limits_{x\to 0}(b^2 tg^2 x)) = \ln(a^2+b^2\cdot\lim\limits_{x\to 0} tg^2 x) = \ln(a^2+b^2\cdot tg^2 0) = \ln a^2$
- $\ln(a^2+b^2 tg^2 0) = \ln a^2$
- $\lim\limits_{x\to\frac{\pi}{2}}(\ln(a^2+b^2 tg^2 x)) = \ln(\lim\limits_{x\to\frac{\pi}{2}}(a^2)+\lim\limits_{x\to\frac{\pi}{2}}(b^2 tg^2 x)) = \ln(a^2+b^2 \lim\limits_{x\to\frac{\pi}{2}} tg^2 x) =$ не существует, а $tg\frac{\pi}{2}$ не определен

Значит ф-ция подинтегральная непрерывна в области $0 \le x \le \frac{\pi}{2}$

Рассматриваем b-как параметр, a-как константу.

Пусть $a>0$ и $b>0$.

$I(a,b)=\int\limits_0^{\frac{\pi}{2}} \ln(a^2+b^2 tg^2 x)\,dx$. Будем брать производную по параметру (по b).

$$\frac{d(\ln(a^2+b^2 tg^2 x))}{db} = \frac{1}{a^2+b^2 tg^2 x}\cdot\frac{d(a^2+b^2 tg^2 x)}{db} = \frac{1}{a^2+b^2 tg^2 x}\cdot 2b\, tg^2 x$$

$$\Rightarrow \frac{\partial I(a,b)}{\partial b} \overset{\text{теория}}{=} \int\limits_0^{\frac{\pi}{2}} \frac{2b\,tg^2 x\,dx}{a^2+b^2 tg^2 x} = \int\limits_0^{\frac{\pi}{2}} \frac{2b\,tg^2 x\,dx}{b^2\left(\frac{a^2}{b^2}+tg^2 x\right)} = \frac{2}{b}\int\limits_0^{\frac{\pi}{2}} \frac{tg^2 x\,dx}{\cos^2 x(1+tg^2 x)\left(\frac{a^2}{b^2}+tg^2 x\right)} =$$

$\cos^2 x\cdot\frac{1}{\cos^2 x}=1$

Замена: $tg\,x = t$

$dt = \frac{1}{\cos^2 x}dx$

Так можно делать, потому что подинтегральная ф-ция непрерывна в области $0\le x\le\frac{\pi}{2}$, $b>0$.

$$= \frac{2}{b}\int\limits_0^{+\infty}\frac{t^2 dt}{(1+t^2)\left(\frac{a^2}{b^2}+t^2\right)}$$

$\Rightarrow$ теория

Решаем через метод неопределенных коэффициентов!

$$\frac{t^2}{(1+t^2)\left(t^2+\frac{a^2}{b^2}\right)} = \frac{At+B}{t^2+\frac{a^2}{b^2}} + \frac{Ct+D}{t^2+1} = \frac{(b^2C+b^2A)t^3+(b^2D+b^2B)t^2+(a^2C+b^2A)t+a^2D+b^2B}{(t^2+1)\left(t^2+\frac{a^2}{b^2}\right)}$$

t^0: $a^2D+b^2B=0 \to B=\frac{-a^2D}{b^2} \to B=\frac{-a^2(1-B)}{b^2} \to B=\frac{-a^2+a^2B}{b^2} \to Bb^2-a^2B=-a^2 \to B=\frac{-a^2}{b^2-a^2}$

t^1: $a^2C+b^2A=0 \to Ca^2=Cb^2 \to C=0,\ A=0$

t^2: $b^2D+b^2B=1 \to D+B=1$

t^3: $b^2C+b^2A=0 \to C+A=0$

$$\Rightarrow \begin{cases} A=0 \\ C=0 \\ B=-\frac{a^2}{b^2-a^2} \\ D=\frac{b^2}{b^2-a^2} \end{cases}$$

$$\frac{\partial I(a,b)}{\partial b} = \frac{2}{b}\int\limits_0^{+\infty}\frac{t^2 dt}{(1+t^2)\left(\frac{a^2}{b^2}+t^2\right)} = \frac{2}{b}\left(-\frac{a^2}{b^2-a^2}\underbrace{\boxed{\int\limits_0^{+\infty}\frac{1}{t^2+\frac{a^2}{b^2}}dt}}_{①} + \frac{b^2}{b^2-a^2}\underbrace{\boxed{\int\limits_0^{+\infty}\frac{1}{t^2+1}dt}}_{②}\right)$$

① (Подстановка:)

$$\frac{\partial I(a,b)}{\partial b} = \frac{2}{b}\int_0^{+\infty}\frac{t^2 dt}{(1+t^2)(\frac{a^2}{b^2}+t^2)} = \frac{2}{b}\left(-\frac{a^2}{b^2-a^2}\boxed{\int_0^{+\infty}\frac{1}{t^2+\frac{a^2}{b^2}}dt} + \frac{b^2}{b^2-a^2}\boxed{\int_0^{+\infty}\frac{1}{t^2+1}dt}\right)$$

1 2

1 $\left[\begin{array}{l}\text{Подстановка:}\\ u = \frac{bt}{a}\\ t = \frac{a}{b}u\\ dt = \frac{a}{b}du\end{array}\right] = \int_0^{+\infty}\frac{a\,du}{b\left(\frac{a^2u^2}{b^2}+\frac{a^2}{b^2}\right)} = \int_0^{+\infty}\frac{b^2 du}{ab(u^2+1)} = \frac{b}{a}\operatorname{arctg} u\Big|_0^{+\infty} = \left\{\begin{array}{l}\text{обратная}\\ \text{замена}\end{array}\right\} = \frac{b}{a}\cdot\operatorname{arctg}\left(\frac{bt}{a}\right)\Big|_0^{+\infty}$

2 $(\operatorname{arctg} t)\Big|_0^{+\infty}$ (табличный)

$$\frac{\partial I(a,b)}{\partial b} = \frac{2}{b}\left(-\frac{a^2}{b^2-a^2}\cdot\frac{b}{a}\cdot\left[(\operatorname{arctg}(\frac{bt}{a}))\Big|_0^{+\infty}\right] + \frac{b^2}{b^2-a^2}\left[(\operatorname{arctg} t)\Big|_0^{+\infty}\right]\right) = \frac{2}{b}\left(-\frac{a^2}{b^2-a^2}\cdot\frac{b}{a}\cdot\left(\frac{\pi}{2}\right)+\frac{b^2}{b^2-a^2}\cdot\left(\frac{\pi}{2}\right)\right) = \frac{-\cancel{2}a^{\cancel{2}}\cancel{b}\,\pi}{\cancel{b}(b^2-a^2)\cancel{a}\cancel{2}} + \frac{\cancel{2}b^{\cancel{2}}\pi}{\cancel{b}(b^2-a^2)\cancel{2}} =$$

$$= \frac{-a\pi + b\pi}{(b^2-a^2)} = \frac{\pi(b-a)}{(b-a)(b+a)} = \frac{\pi}{b+a}$$

$I(a,b) = \int\frac{\partial I(a,b)}{\partial b}db = \int\frac{\pi}{b+a}db = \pi\ln(a+b)+C(a)$. Найдём $C(a)$, не зависящую от b.

Для этого приближаем b к нулю: $I(a,b) = \int_0^{\frac{\pi}{2}}\lim_{b\to 0}(\ln(a^2+b^2\operatorname{tg}^2 x))dx = \int_0^{\frac{\pi}{2}}\ln a^2 dx = \pi\ln a$

$$I(a,b) = \int\lim_{b\to 0}\frac{\pi}{b+a}db = \int\frac{\pi}{a}db = \pi\ln a + C(a)$$

Тогда $\pi\ln a = \pi\ln a + C(a) \to C(a) = 0$. Решение при $a>0$, $b>0$: $I(a,b) = \pi\ln(a+b)$

Это же решение применимо для $a<0$ и $b<0$, если записать a и b под знаком модуля.

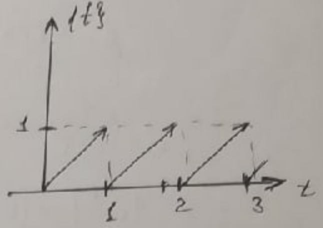
$I(a,b) = \int_0^{\frac{\pi}{2}}\ln(a^2+b^2\operatorname{tg}^2 x)dx = \int_0^{\frac{\pi}{2}}\ln(|a|^2+|b|^2\operatorname{tg}^2 x)dx = I(|a|,|b|) = \pi\ln(|a|+|b|)$

Ответ: $I(a,b) = \pi\ln(|a|+|b|)$

8. Применяя свойства преобразования Лапласа, найти изображение функции
$f(t) = A\{t\}$, где $\{t\}$ — дробная часть.

Для выполнения условий по преобразованию Лапласа → $f(t)=0$ при $t<0$, поэтому определим функцию, как:

$$f(t) = \begin{cases} A\{t\}, & t>0 \\ 0, & t<0 \end{cases} \Rightarrow f(t)\,\eta(t) \text{ — оригинал}$$



Пользуемся теоремой запаздывания:

$$f(t-\tau) \risingdotseq e^{-\tau p} F(p), \quad \forall \tau > 0$$

$$\Rightarrow f(t) = \begin{cases} t, & t\in[0,1) \\ t-1, & t\in[1,2) \\ t-2, & t\in[2,3) \\ \dots \\ t-N, & t\in[N-1,N) \end{cases}$$

Чтобы отдельно представить ф-ии на интервалах → используем функцию Хэвисайда:

$$t_{[0,1)} = (\eta(t)-\eta(t-1))t$$

$$(t-1)_{[1,2)} = (\eta(t-1)-\eta(t-2))\cdot(t-1)$$

$$\dots$$

$$\Rightarrow f(t) = (\eta(t)-\eta(t-1))\,t + (\eta(t-1)-\eta(t-2))\cdot(t-1) + (\eta(t-2)-\eta(t-3))\cdot(t-2) + (\eta(t-3)-\eta(t-4))\cdot(t-3)+\ldots = \eta(t)\cdot t + \eta(t-1)\cdot(t-1-t) + \eta(t-2)(t-2-t+1) + \eta(t-3)(t-3-t+2)+\ldots =$$

$$= \eta(t)\cdot t - \eta(t-1) - \eta(t-2) - \eta(t-3) - \ldots = \{\text{далее по теореме запаздывания}\} \risingdotseq$$

$$= \frac{1}{p^2} - \frac{e^{-p}}{p} - \frac{e^{-2p}}{p} - \frac{e^{-3p}}{p} - \ldots = \frac{1}{p^2} + \frac{1}{p}\left(e^{-p}+e^{-2p}+e^{-3p}+\ldots\right) =$$

?

$$= \frac{1}{p^2} - \frac{1}{p}\cdot\frac{e^{-p}}{1-e^{-p}} = \frac{1}{p^2} - \frac{1}{p}\cdot\frac{1}{e^{p}-1}$$

$$\Rightarrow \quad A\{t\} \risingdotseq \frac{A}{p^2} - \frac{A}{p}\cdot\frac{1}{e^{p}-1} = \frac{A}{p}\left(\frac{1}{p} - \frac{1}{e^{p}-1}\right)$$

$|q|<1 \qquad p = s + i\nu$

$\operatorname{Re} p = s > s_0 \qquad s_0 = 0$

$$|e^{-p}| = |e^{-s}|\cdot\underbrace{|e^{-i\nu}|}_{=1} < 1$$

10. Применяя дифференцирование или интегрирование по параметру, вычислить несобственный интеграл:

$$I=\int_0^\infty e^{-ax}\,\frac{\sin^2 mx}{x}\,dx \qquad (a>0)$$

$I(m)=\int_0^\infty \frac{e^{-ax}}{x}\sin^2 mx\,dx$

$I'(m) \overset{?}{=} \int_0^\infty \frac{e^{-ax}}{\cancel{x}}\cdot 2\sin mx\cdot\cos mx\cdot\cancel{x}\,dx = \int_0^\infty e^{-ax}\sin 2mx\,dx = \left\{\begin{array}{l} u=\sin 2mx \to du = 2m\cos 2mx\,dx \\ dv=e^{-ax}dx \to v=-\frac{1}{a}e^{-ax}\end{array}\right\} =$

$= \underbrace{-\frac{e^{-ax}}{a}\cdot\sin 2mx\Big|_0^\infty}_{=0} + \frac{2m}{a}\int_0^\infty e^{-ax}\cos 2mx\,dx = \left\{\begin{array}{l} u=\cos 2mx \to du=-2m\sin 2mx\,dx \\ dv=e^{-ax}dx\to v=-\frac1a e^{-ax}\end{array}\right\}=$

$=\frac{2m}{a}\left[-\frac1a e^{-ax}\cdot\cos 2mx\Big|_0^\infty - \frac{2m}{a}\int_0^\infty e^{-ax}\sin 2mx\,dx\right] = \frac{2m}{a}\left[\frac1a - \frac{2m}{a}\int_0^\infty e^{-ax}\sin 2mx\,dx\right]$

$\Rightarrow \int_0^\infty e^{-ax}\sin 2mx\,dx = \frac{2m}{a^2}-\frac{4m^2}{a^2}\int_0^\infty e^{-ax}\sin 2mx\,dx$

$\int_0^\infty e^{-ax}\sin 2mx\,dx\cdot\left[1+\frac{4m^2}{a^2}\right] = \frac{2m}{a^2}$ $\qquad \sin\omega t \doteqdot \frac{\omega}{\omega^2+p^2}$

$\int_0^\infty e^{-ax}\sin 2mx\,dx = \frac{2m\cdot a^2}{(4m^2+a^2)a^2}=\frac{2m}{4m^2+a^2}$ $\quad *$

$\Rightarrow I'(m)=\frac{2m}{4m^2+a^2}$

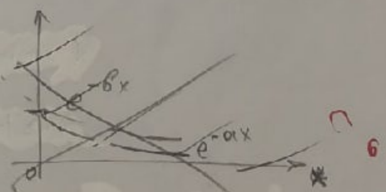
$I(m)=\int\frac{2m\,dm}{4m^2+a^2}=\int\frac{d(m^2)}{4m^2+a^2}=\frac14\int\frac{d(4m^2)}{4m^2+a^2}=\frac14\ln|4m^2+a^2|+C(a)$

~ модуль можно убрать, $\forall m, a \to |\dots|>0$

$I(m=0)=0=\frac14\ln a^2 + C(a) \to C(a) = -\frac14\ln a^2$

$\Rightarrow I(m)=\frac14\left[\ln(4m^2+a^2)-\ln a^2\right]=\frac14\ln\left(\frac{4m^2+a^2}{a^2}\right)$

(?) $\int_0^\infty e^{-ax}\sin 2mx\,dx$ ~ сходится?



$|\sin 2mx|\le 1$ ~ ограничена.

$\int_0^\infty e^{-ax}dx$ ~ сходится?

$e^{-ax}=\frac{1}{e^{ax}}<\frac{1}{e^{bx}}=e^{-bx}$, т.е. будем подбирать такое $b=\text{const}$, что выполняется неравенство на $[0,\infty)$

$\int_0^\infty e^{-bx}dx = -\frac1b e^{-bx}\Big|_0^\infty=\frac1b$ => сходится по Вейерштрассу

$\Rightarrow \int_0^\infty e^{-ax}\sin 2mx\,dx$ ~ сходится по Абелю

$\int_0^\infty e^{-ax}dx=-\frac1a e^{-ax}\Big|_0^\infty=\frac1a$ ~ сх-ся $(a>0)$ => сходится по Абелю

9. Вычислить несобственный интеграл, выразив его через Эйлеровы:

$$\int_{-\frac{\pi}{4}}^{\frac{\pi}{4}} \left(\frac{\cos x+\sin x}{\cos x-\sin x}\right)^{\cos 2\alpha} dx$$

$$\int_{-\frac{\pi}{4}}^{\frac{\pi}{4}} \left(\frac{\cos x+\sin x}{\cos x-\sin x}\right)^{\cos 2\alpha} dx = \left\{ \begin{array}{l} \frac{\cos x+\sin x}{\cos x-\sin x} = t \rightarrow \frac{(-\sin x+\cos x)(\cos x-\sin x)-(-\sin x-\cos x)(\cos x+\sin x)}{(\cos x-\sin x)^2}dx = dt \\ \frac{(\cos x-\sin x)^2+(\cos x+\sin x)^2}{(\cos x-\sin x)^2}dx = dt \rightarrow (1+t^2)dx = dt \rightarrow dx = \frac{dt}{1+t^2} \end{array} \right\} =$$

$$= \int_0^{\infty} t^{\cos 2\alpha}\cdot\frac{dt}{1+t^2} = \left\{ \begin{array}{l} t^2 = u\ ;\ t=\sqrt{u},\ t^{\cos 2\alpha} = u^{\frac{\cos 2\alpha}{2}} \\ 2t\,dt = du \rightarrow dt = \frac{du}{2\sqrt{u}} \end{array} \right\} =$$

$$= \int_0^{\infty} u^{\frac{\cos 2\alpha}{2}}\cdot\frac{1\cdot du}{2\sqrt{u}(1+u)} = \frac{1}{2}\int_0^{\infty}\frac{u^{\frac{\cos 2\alpha-1}{2}}}{1+u}du = \left\{ \begin{array}{l} p-1 = \frac{\cos 2\alpha}{2}-\frac{1}{2} \rightarrow p = \frac{\cos 2\alpha+1}{2} \\ p+q=1 \rightarrow q = 1-p = \frac{1-\cos 2\alpha}{2} \end{array} \right\} =$$

$$= \frac{1}{2}B\left(\frac{\cos 2\alpha+1}{2}, \frac{1-\cos 2\alpha}{2}\right) \ominus$$

$$\cos^2\alpha = \frac{1+\cos 2\alpha}{2}$$

$$B(p, 1-p) = \frac{\pi}{\sin \pi p}$$

$$\ominus\ \frac{\pi}{2}\cdot\frac{1}{\sin\left(\frac{\pi}{2}(\cos 2\alpha+1)\right)} = \frac{\pi}{2}\cdot\frac{1}{\cos\left(\frac{\pi}{2}\cos 2\alpha\right)} = \frac{\pi}{2}\cdot\frac{1}{\cos\left(\frac{\pi}{2}(2\cos^2\alpha-1)\right)} = \frac{\pi}{2\sin(\pi\cos^2\alpha)}$$

$$\int_{-\frac{\pi}{4}}^{\frac{\pi}{4}} \left(\frac{\cos x+\sin x}{\cos x-\sin x}\right)^{\cos 2\alpha} dx = \frac{\pi}{2}\cdot\frac{1}{\sin\left(\frac{\pi}{2}(\cos 2\alpha+1)\right)} = \frac{\pi}{2\sin(\pi\cos^2\alpha)}$$

Область существования: $\begin{cases} p>0 \\ q>0 \end{cases} \Rightarrow \begin{cases} \frac{\cos 2\alpha+1}{2}>0 \\ \frac{1-\cos 2\alpha}{2}>0 \end{cases} \quad \begin{cases} \cos 2\alpha+1>0 \quad (1) \\ 1-\cos 2\alpha>0 \quad (2) \end{cases}$

(1): $\cos 2\alpha > -1 \Rightarrow \cos 2\alpha \neq -1$

$2\alpha \neq \pi + 2\pi k,\ k\in\mathbb{Z} \rightarrow \alpha \neq \frac{\pi}{2}+\pi k,\ k\in\mathbb{Z}$

(2): $1-\cos 2\alpha > 0 \Rightarrow \cos 2\alpha < 1$

$\cos 2\alpha \neq 1$

$2\alpha \neq \pi k,\ k\in\mathbb{Z} \rightarrow \alpha \neq \frac{\pi k}{2},\ k\in\mathbb{Z}.$

$$\Rightarrow \left[\begin{array}{l} \alpha \neq \frac{\pi k}{2} \\ \alpha \neq \frac{\pi}{2}+\pi k \end{array} \right., \ k\in\mathbb{Z}$$

N3(5) $F(p) = \frac{1}{\sqrt{p}} e^{-a/p} =$ $\left\{ e^z = \sum_{k=0}^{\infty} \frac{z^k}{k!} \right\}$

$= \frac{1}{p^{1/2}} \cdot \sum_{k=0}^{\infty} \frac{\left(-\frac{a}{p}\right)^k}{k!} =$ $\left\{ k! = \Gamma(k+1) \right\}$ $\left\{ \frac{1}{p^k} \risingdotseq \frac{t^{k-1}}{(k-1)!} = \frac{t^{k-1}}{\Gamma(k)} \right\}$

$= \frac{1}{p^{1/2}} \sum_{k=0}^{\infty} \frac{(-1)^k a^k}{k!} \frac{1}{p^k} =$

$= \sum_{k=0}^{\infty} \frac{(-1)^k a^k}{k!} \frac{1}{p^{k+1/2}} \risingdotseq \sum_{k=0}^{\infty} \frac{(-1)^k a^k}{k!\, \Gamma(k+\frac{1}{2})} t^{k-\frac{1}{2}} =$

$= \frac{1}{\sqrt{t}} \sum_{k=0}^{\infty} \frac{(-1)^k (at)^k}{\Gamma(k+1)\Gamma(k+\frac{1}{2})} = \frac{1}{\sqrt{t}} \sum_{k=0}^{\infty} \frac{(-1)^k (\sqrt{at})^{2k}}{\Gamma(k+1)\Gamma(k+\frac{1}{2})} =$

$\left\{ \cos z = \sum_{k=0}^{\infty} \frac{(-1)^k z^{2k}}{(2k)!} \right\}$

Формула умножения Гаусса:

$\Gamma(z) \cdot \Gamma(z+\frac{1}{n}) \cdots \Gamma(z+\frac{n+1}{n}) = n^{\frac{1}{2}-nz} \cdot (2\pi)^{\frac{n-1}{2}}$

$n = 2$ (получена Лежандром)

$\Gamma(z)\Gamma(z+\frac{1}{2}) = 2^{1-2z} \cdot \sqrt{\pi}\, \Gamma(2z)$

$z = k+\frac{1}{2} \Rightarrow \Gamma(k+1)\Gamma(k+\frac{1}{2}) =$

$= 2^{1-2(k+\frac{1}{2})} \cdot \sqrt{\pi}\, \Gamma(2(k+\frac{1}{2})) =$

$= 2^{\cancel{1}-2k-\cancel{1}} \cdot \sqrt{\pi}\, \Gamma(2k+1) = 2^{-2k} \cdot \sqrt{\pi} \cdot \Gamma(2k+1)$

$= \frac{1}{\sqrt{t}} \sum_{k=0}^{\infty} \frac{(-1)^k (\sqrt{at})^{2k}}{\sqrt{\pi} \frac{1}{2^{2k}} \Gamma(2k+1)} = \frac{1}{\sqrt{\pi}\sqrt{t}} \sum_{k=0}^{\infty} \frac{(-1)^k (2\sqrt{at})^{2k}}{(2k)!} =$

$= \frac{1}{\sqrt{\pi}\sqrt{t}} \cos(2\sqrt{at})$

$$\int_{-1}^{1} \frac{(1+x)^{2m-1}(1-x)^{2n-1}\,dx}{(1+x^2)^{m+n}} = \left\{ \begin{matrix} y=\frac{x+1}{2} & x=2y-1 \\ dx=2dy & 1-x=1-2y+1=2(1-y) \end{matrix} \right\} =$$

$$= \int_0^1 \frac{(2\cdot y)^{2m-1}(2(1-y))^{2n-1}}{(1+(2y-1)^2)^{m+n}}\,dy = 2^{2m-1}\cdot 2^{2n-1}\int_0^1 \frac{y^{2m-1}(1-y)^{2n-1}\,dy}{(1+(2y-1)^2)^{m+n}} =$$

$$= \left\{ \begin{matrix} \frac{1}{y}-1=t & y=\frac{1}{t+1} & dy=-\frac{dt}{(t+1)^2} \\ \frac{1}{y}=t+1 & 1-y=1-\frac{1}{t+1}=\frac{t}{t+1} & \end{matrix} \right\} =$$

$$= + 2^{2(m+n)-2}\int_0^\infty \frac{t^{2n-1}\,dt}{(t+1)^{2m-1}(t+1)^{2n-1}(t+1)^2\left(1+\left(2\frac{1}{t+1}-1\right)^2\right)^{m+n}} =$$

$$= 2^{2(m+n)-2}\int_0^\infty \frac{t^{2n-1}}{(t+1)^{2(m+n)}\left(\frac{(t+1)^2}{(t+1)^2}+\frac{(1-t)^2}{(t+1)^2}\right)^{m+n}}\,dt =$$

$$= 2^{2(m+n)-2}\int_0^\infty \frac{t^{2n-1}}{((t+1)^2+(1-t)^2)^{m+n}}\,dt = \frac{2^{2(m+n)-2}}{2\cdot 2^{m+n}}\int_0^\infty \frac{t^{2(n-1)}\,dt^2}{(t^2+1)^{m+n}} \;(=)$$

$$t^2+2t+1+1-2t+t^2 = 2t^2+2 = 2(t^2+1)$$

$$\frac{t^{2n-1}\cdot t}{t} = t^{2(n-1)}\cdot t$$

$$(=)\; \frac{2^{2(m+n)-2}}{2^{m+n+1}}\int_0^\infty \frac{(t^2)^{n-1}}{(t^2+1)^{m+n}}\,dt^2 = 2^{m+n-3}\,B(n,m)$$

$$n>0,\ m>0$$

$$\frac{t^2}{a^2+b^2t^2}\cdot\frac{1}{1+t^2} = \frac{t^2}{a^2+b^2t^2+t^2a^2+b^2t^4}$$

Доказательство сходимости равномерной.

$$I = \int_0^\infty \frac{2bt^2}{a^2+b^2t^2}\cdot\frac{1}{1+t^2}\,dt = \frac{2}{b}\int_0^\infty \frac{t^2}{\left(\frac{a^2}{b^2}+t^2\right)(1+t^2)}$$

$$|f(x,y)| = \left|\frac{t^2}{\left(\frac{a^2}{b^2}+t^2\right)(1+t^2)}\right|$$

$$f(x,y) \le \frac{1}{1+t^2}$$

$$\int_0^\infty \frac{1}{1+t^2} = \operatorname{arctg} t\Big|_0^\infty = \frac{\pi}{2}$$ – сходится.

Следовательно интеграл равномерно сходится равномерно относительно "b" по признаку Вейерштрасса.

$$I(\alpha)=\int_0^{\pi/2}\ln(1+\alpha\sin^2 x)\cos^2 x\,dx \qquad (\alpha>-1)$$

функция четная

$$\dot I(\alpha)=\int_0^{\pi/2}\frac{\sin^2x\cos^2x}{1+\alpha\sin^2x}dx=\int_0^{\pi/2}\frac{\sin^2x\cos^2x}{(ctg^2x+(\alpha+1))\sin^2x}dx\overset{d(ctg\,x)}{=}\frac{1}{2}\int_{-\infty}^{+\infty}\frac{t^2\,dt}{(t^2+1)^2(t^2+(\alpha+1))}\ \ominus$$

$$\Big\{\ t=ctg\,x \quad -\frac{dx}{\sin^2x}=dt \quad \sin^2x=\frac{1}{1+t^2} \quad \cos^2x=\frac{t^2}{1+t^2}\ \Big\}$$

$$res\ \frac{1}{1!}\lim_{t\to i}\frac{d}{dt}\left((t-i)^m f(t)\right) \qquad res\,f=\lim_{t\to i}(t-i)f(t)$$

$$\ominus\ \frac{1}{2}\int_{-\infty}^{+\infty}\frac{t^2\,dt}{(t-i)^2(t+i)^2(t-i\sqrt{\alpha+1})(t+i\sqrt{\alpha+1})}$$

Полюс 1-го порядка

$$I'(\alpha)=\frac{2\pi i}{2}\left[Res[f,i]+Res[f,i\sqrt{\alpha+1}]\right]=\frac{\pi}{4}\frac{\alpha+2}{\alpha^2}-\frac{2\pi}{4\alpha^2}\sqrt{\alpha+1}=$$

полюс 2-го

$$=\frac{\pi}{4\alpha^2}\left(\alpha+2-2\sqrt{\alpha+1}\right)$$

$$I(\alpha)=\frac{\pi}{4}\int\frac{\alpha+2-2\sqrt{\alpha+1}}{\alpha^2}d\alpha+C=\frac{\pi}{4}\ln|\alpha|-\frac{2\pi}{4\alpha}+\frac{\pi\sqrt{\alpha+1}}{2\alpha}-\frac{\pi}{4}\ln|\sqrt{\alpha+1}-1|+\frac{\pi}{4}\ln|\sqrt{\alpha+1}+1|+C$$

$$I(\alpha)=\frac{\pi}{2}\left(\frac{\sqrt{\alpha+1}-1}{\alpha}-\frac{1}{2}\ln|\sqrt{\alpha+1}-1|+\frac{1}{2}\ln|\sqrt{\alpha+1}+1|+\frac{1}{2}\ln|\alpha|\right)+C$$

$$I(\alpha)=\frac{\pi}{2}\left(\frac{\sqrt{\alpha+1}-1}{\alpha}+\ln|\sqrt{\alpha+1}+1|\right)+C$$

$$I(\alpha_0)\Big|_{\alpha=0}=\int g(\alpha)\,d\alpha\Big|_{\alpha=\alpha_0}=\lim_{\alpha\to 0}\frac{\pi}{2}\left(\frac{\sqrt{\alpha+1}-1}{\alpha}+\ln|\sqrt{\alpha+1}+1|\right)+C\ \Rightarrow\ -\frac{\pi}{2}\left(\frac{1}{2}+\ln 2\right)=C$$

$$I(\alpha)=\frac{\pi}{2}\left(\frac{\sqrt{\alpha+1}-1}{\alpha}+\ln|\sqrt{\alpha+1}+1|-\frac{1}{2}-\ln 2\right)=\frac{\pi}{2}\left(\frac{\sqrt{\alpha+1}-1}{\alpha}+\ln\left|\frac{\sqrt{\alpha+1}+1}{2}\right|-\frac{1}{2}\right)$$

?

$\frac{\sin^2x\cos^2x}{1+\alpha\sin^2x}>0$ ~ функция положительная

т.к. $\alpha>-1$ → $\frac{1}{1+\alpha\sin^2x}>0$

Дини:
- непрерывность
- положит

$$f(x,\alpha)=\frac{\sin^2x\cos^2x}{1+\alpha\sin^2x}>0$$

класс непр (знаменат. не обнул.)

$$G(\alpha)=\int_0^{\pi/2}\frac{\sin^2x\cos^2x}{1+\alpha\sin^2x}dx=\frac{\pi}{4}\frac{(\alpha+2-2\sqrt{\alpha+1})}{\alpha^2}=\lim_{\alpha\to0}\frac{\pi}{4}\frac{(\alpha+2-2\sqrt{\alpha+1})}{\alpha^2}=\frac{\pi}{16}$$

~ функция непрерывная; непрерывная

— Сходится по Дини.

Если $f(x,y)$ непр. в Π_α, и $f(x,y)\geq 0$ в нем, а $I(y)=\int_a^\omega f(x,y)\,dx$ — непр. на $[c,d]$, то $\int f(x,y)\,dx$ равном. сход. относ. y на $[c,d]$

№9 $\sqrt{9}$

$$\int_0^\infty \frac{e^{-ax^2}}{x^2+b^2}\,dx = I(\alpha,b)$$

$$\int_0^\infty f(x)\,G(x)\,dx = \int_0^\infty g(y)\,F(y)\,dy \qquad \begin{array}{l} f(t) \doteqdot F(p) \\ G(p) \doteqdot g(t) \end{array}$$

$$\left\{ y=x^2 \quad dy = 2x\,dx \right\} \Rightarrow I(\alpha,\beta) = \frac{1}{2}\int_0^\infty \frac{e^{-ay}}{\sqrt{y}\,(y+b^2)}\,dy$$

(?) $$\frac{1}{\sqrt{y}} \doteqdot \sqrt{\frac{\pi}{x}} \qquad (3)\ \frac{e^{-ay}}{y+b^2} \doteqdot e^{-b^2(t-a)}\,\eta(t-a)$$

$$I(a,b) = \int_0^\infty e^{-b^2(x-a)}\,\eta(x-a)\,\frac{\sqrt{\pi}}{\sqrt{x}}\,dx \Rightarrow$$

$$\left\{ \lambda = b^2\sqrt{x} \qquad d\lambda = \frac{b^2\,dx}{2\sqrt{x}} \right\}$$

$$\Rightarrow \int_0^\infty e^{-b^2(x-a)}\,\eta(x-a)\cdot\frac{\sqrt{\pi}}{\sqrt{x}}\,dx = \int_{b\sqrt{a}}^\infty \frac{\sqrt{\pi}}{b}\,e^{-\lambda^2}e^{ab^2}\,d\lambda$$

$$I(\alpha,b) = \frac{\sqrt{\pi}}{b}\int_{b\sqrt{a}}^\infty e^{-\lambda^2}e^{ab^2}\,d\lambda$$

$$\operatorname{erf} x = \frac{2}{\sqrt{\pi}}\int_0^x e^{-t^2}\,dx \longrightarrow \underbrace{1-\operatorname{erf} x = \frac{2}{\sqrt{\pi}}\int_x^\infty e^{-t^2}\,dt}$$

$$I(a,b) = \frac{\pi}{2b}e^{ab^2}\left(1-\operatorname{erf} b\sqrt{a}\right) = \frac{\pi}{2b}e^{ab^2}\operatorname{erfc}(b\sqrt{a})$$

(?) $$\underbrace{\frac{1}{\sqrt{y}} = y^{-\frac{1}{2}} \doteqdot \frac{\Gamma(-\frac{1}{2}+1)}{p^{-\frac{1}{2}+1}} = \frac{\Gamma(\frac{1}{2})}{p^{1/2}} = \frac{\sqrt{\pi}}{\sqrt{p}}}$$ Dok: $F(p) = \int_0^\infty f(t)e^{-pt}dt$

$$\int_0^\infty y^{-\frac{1}{2}} e^{-pz}\,dz = \left\{ yp = z \to y = \frac{z}{p} \quad dy = \frac{dz}{p} \right\} = \frac{1}{p}\int_0^\infty p^{1/2}\,z^{-\frac{1}{2}}\,e^{-z}\,dz \Rightarrow$$

$$\left\{ p'-1 = -\frac{1}{2} \quad \to p' = 1-\frac{1}{2} \right\} \Rightarrow \frac{1}{p^{1-1/2}}\,\Gamma(1-1/2) = \frac{\sqrt{\pi}}{\sqrt{p}}$$

$\int_{-\infty}^{+\infty} \frac{\operatorname{sh}\alpha x}{\operatorname{sh}\beta x}dx = 2\int_0^{+\infty} \frac{e^{\alpha x}-e^{-\alpha x}}{e^{\beta x}-e^{-\beta x}}dx = \left\{ e^{-x}=t,\ x=-\ln t,\ dx=-\frac{dt}{t}\right\} =$ № 2 (3864)

$= 2\int_1^0 \frac{t^{-\alpha}-t^{\alpha}}{t^{-\beta}-t^{\beta}}\cdot\frac{dt}{t} = 2\int_0^1 t^{\beta-\alpha-1}\frac{(1-t^{2\alpha})}{(1-t^{2\beta})}dt = \left\{t^{2\beta}=u,\ u^{1/2\beta}=t\right\}$ $dt=\frac{1}{2\beta}u^{\frac{1-2\beta}{2\beta}}du$ $\beta<0?$

$=2\int_0^1 \frac{1}{2\beta}u^{\frac{\beta-\alpha-1}{2\beta}}\frac{1-u^{\frac{\alpha}{\beta}}}{1-u}\cdot u^{\frac{1-2\beta}{2\beta}}du = \frac{1}{\beta}\int_0^1 u^{-\frac{\beta-\alpha}{2\beta}}\frac{(1-u^{\frac{\alpha}{\beta}})}{(1-u)}du =$

$\frac{\beta-\alpha-1}{2\beta}+\frac{1-2\beta}{2\beta}=-\frac{\beta+\alpha}{2\beta}$

$\frac{1}{\beta}\int_0^1 \frac{u^{-\frac{\beta-\alpha}{2\beta}}-u^{\frac{\alpha-\beta}{2\beta}}}{(1-u)^{1+\varepsilon}}du$ $\int_0^1\frac{x^{p-1}-x^{-p}}{1-x}$

№ 3866 $p=\frac{\beta-\alpha}{2\beta}$ $p=\frac{\alpha+\beta}{2\beta}$

$= \frac1\beta\int_0^1 \frac{u^{-\frac{\beta-\alpha}{2\beta}}-u^{\frac{\alpha-\beta}{2\beta}}}{(1-u)^{-\xi+1}}du = \frac1\beta\lim_{\xi\to0}\int_0^1\frac{u^{-\frac{\beta-\alpha}{2\beta}}}{(1-u)^{1-\xi}}du - \frac1\beta\int_0^1\frac{u^{\frac{\alpha-\beta}{2\beta}}}{(1-u)^{1-\xi}}du = \frac1\beta\lim_{\xi\to0}[B(p,\xi)-B(1-p,\xi)]$

$\left(-\frac{\beta-\alpha}{2\beta}=p-1,\quad \frac{\alpha-\beta}{2\beta}=-p\right)$

$\frac1\beta\lim_{\xi\to0}\frac{\Gamma(\xi)[\Gamma(p)\Gamma(1-p+\xi)-\Gamma(1-p)\Gamma(p+\xi)]}{\Gamma(p+\xi)\Gamma(1-p+\xi)} = \lim_{\xi\to0}\frac{1}{\beta\Gamma(p+\xi)\Gamma(1-p+\xi)\Gamma(1-\xi)}\lim_{\xi\to0}\Gamma(\xi)\Gamma(1-\xi)\left(\Gamma(p)\Gamma(1-p+\xi)-\Gamma(1-p)\Gamma(p+\xi)\right)=$

$=\frac{1}{\beta\Gamma(p)\Gamma(1-p)\Gamma(1)}\lim_{\xi\to0}\Gamma(\xi)\Gamma(1-\xi)[\Gamma(p)\Gamma(1-p+\xi)-\Gamma(1-p)\Gamma(p+\xi)] = \frac{\sin\pi p}{\beta}\lim_{\xi\to0}\frac{\Gamma(p)\Gamma(1-p+\xi)-\Gamma(1-p)\Gamma(p+\xi)}{\sin\pi\xi}=$

$=\frac{\sin\pi p}{\beta}\lim_{\xi\to0}\frac{\Gamma(p)\Gamma'(1-p+\xi)-\Gamma(1-p)\Gamma'(p+\xi)}{\pi\cos\pi\xi}=\frac{\sin\pi p}{\beta\pi}[\Gamma(p)\Gamma'(1-p)-\Gamma(1-p)\Gamma'(p)] =$

$\| \ \Gamma(p)\Gamma'(1-p)-\Gamma(1-p)\Gamma'(p) = -\frac{d}{dp}\left(\Gamma(p)\Gamma(1-p)\right) = -\frac{d}{dp}\left(\frac{\pi}{\sin p\pi}\right) = \frac{\pi^2\cos p\pi}{\sin^2 p\pi}$

$= \frac{\sin\pi p}{\beta\pi}\cdot\frac{\pi^2\cos p\pi}{\sin^2 p\pi}=\frac{\pi}{\beta}\operatorname{ctg}p\pi=\frac{\pi}{\beta}\operatorname{ctg}\left(\pi\frac{\beta-\alpha}{2\beta}\right)=\frac{\pi}{\beta}\operatorname{ctg}\left(\frac{\pi}{2}-\frac{\pi\alpha}{2\beta}\right)=\frac{\pi}{\beta}\operatorname{tg}\left(\frac{\pi\alpha}{2\beta}\right)$ $\beta>0$, $\beta>|\alpha|$

, где $p=\frac{\beta-\alpha}{2\beta}$ → $0<|\alpha|<\beta$ — при $\beta<0$ $|\alpha|<$ отриц. числа $\varnothing$ → $\beta>0$

$0<p<1$

(?) 1) $f(x)=x^{p-1}-x^{-p}$ $g(x)=(1-x)$ $\lim_{x\to1}\frac{x^{p-1}-x^{-p}}{(1-x)}=-2p+1$ — предел сх-ся ($0<p<1$)

2) $\frac{|x^{p-1}-x^{-p}|}{(1-x)^{1-\xi}}\le\frac{|x^{p-1}-x^{-p}|}{|1-x|}$ — сх-ся этот → сх-ся $\int_0^1\frac{(x^{p-1}-x^{-p})}{(1-x)^{1-\xi}}$